"TWENTY PERCENT FREE!"

So how much does the original bar weigh?

NGĀREWA HĀWERA and **MERILYN TAYLOR** describe a rich task based on a real world situation that stimulated a great deal of mathematical reasoning. Their research on how students attempted the task is most revealing.

Introduction

Developing critical numeracy is important in a society where mathematics plays a particular and significant role (Stoessiger, 2002). One way of helping learners to develop the level of numeracy required to participate in society is by exploring ideas embedded in rich, accessible tasks (Ahmed, 1987). These can be linked to contexts that have relevance in their lives.

One mathematics idea widely employed in everyday living is percentages. People want and need to make informed decisions about, for example, purchasing an item, education, health, or employment-related issues. All of these contexts can involve the need for an understanding of percentages in order to compare items or situations.

A common context for exposure to percentages is where people act as consumers. Percentages are often presented to entice and sway people to make decisions about the purchase of a particular product or service. Understanding percentages and what they represent is therefore critical for appreciating the consequences of any decisions that are made.

Percentages

One mathematical perspective about percentages is fractional where the percent can be used to describe part of a whole. This means that the whole will be 100% and any percentage a part of that whole. Another idea that illustrates percentages describes a proportional relationship between two quantities. Percentages that compare two quantities in a ratio sense can describe a change in a set or object. If an object costs $120 and has a rise in cost of 25%, the new price will be $150 (Parker & Leinhardt, 1995).

Understanding and learning the various ideas surrounding percentages has proven to be a challenge for many. Learners need to make connections to fractions, decimal, ratios and proportions (Parker & Leinhardt, 1995; Reys, Lindquist, Lambdin & Smith, 2007). They need to understand that percentages have to be considered in relation to the context in which the problem exists (White & Mitchelmore, 2005). A reliance on the fractional idea of a percent representing part of a whole can create difficulties for students who then find it problematical to comprehend a percentage greater than 100% (Parker & Leinhardt, 1995).

The concise notation of percentage can also be misleading for learners who may not understand the need to unravel all aspects of a question or statement when given a mathematical problem to solve. The use of the percentage sign (%) often means that fewer words may be used to express an idea (e.g., "one and a half times the size of…" becomes "150% of…"). Understanding a percentage expression fully is fundamental to solving problems where a percentage change has occurred. Unless learners are helped to consider a broader perspective of percentage and the possible meanings that can be involved, they will struggle to understand the implications percentages can demand in a range of situations (Parker & Leinhardt, 1995).

This study

This research was conducted with a class of 27 students who had spent four hours revising ideas about percentages in a range of contexts. Their exploration included identifying strategies for calculation of percentages. The students spent about two hours in small groups (2–4) working on this task. The learning environment in the class encouraged participation and collaborative work. Students were expected to justify their thinking and share ideas publicly.

Exploring percentages

One way of exploring percentages is by examining readily available supermarket products that proclaim greater value or a larger quantity for the same dollar amount. This is a common manufacturing and marketing strategy for encouraging people to purchase a particular product.

The task explored here links to the context of buying a chocolate bar which indicates that the purchaser would be getting 20% more than the original bar, for the same price as the original one (see Figure 1). Students were shown the wrapping of a chocolate bar weighing 300 grams. The manufacturer claimed that it included 20% more than the original bar. Students were asked to find out how much the original bar weighed according to this information. This is the reversal of the more commonly presented problem where learners are given a specific quantity or amount and asked to find the new amount arising from a particular percentage increase.

The students' immediate response was to divide the 300 grams by 5 because 20% was considered to be equivalent to one fifth. This misconception is reflected in Figure 2 when the students were asked to pictorially represent their thinking. Upon determining the answer to be 60 grams, they then deducted this from 300 grams and announced that the original bar weighed 240 grams! When asked

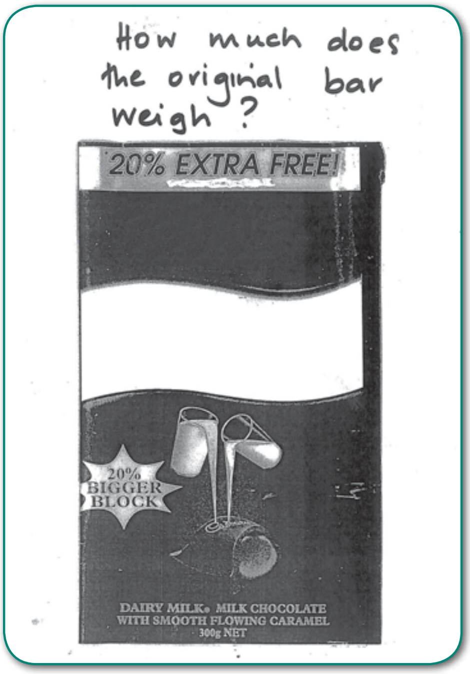

Figure 1

how they might check their mathematical thinking and reasoning, the students eventually realised that if they found 20% of 240 grams and added that on to 240, it should equate to 300 grams (i.e., the weight of the chocolate bar that incorporated the 20% extra)! They proceeded to follow this line of reasoning and were concerned to find that 20% of 240 grams was 48 grams. Adding this amount to 240 grams gave a total of 288 grams—not what they expected!

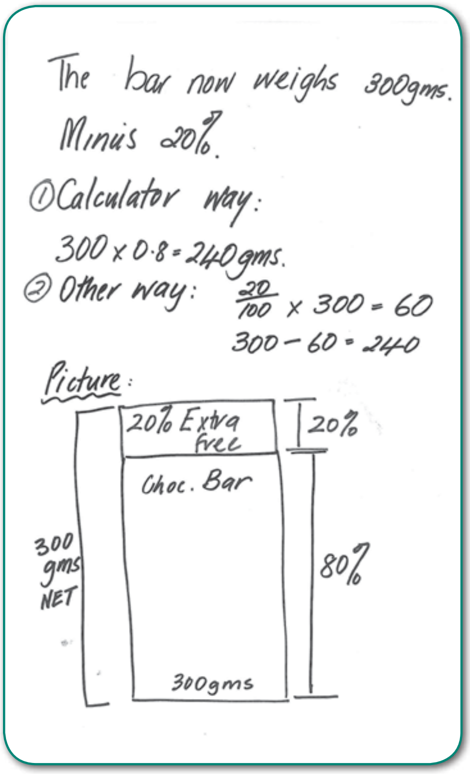

Figure 2

Working with percentages greater than 100% is problematic for those students who have learned that the quantity before them (whatever that might be) represents 100%. In this particular example it was immediately accepted by the students that the given chocolate bar represented one whole, which was therefore equal to 100% (not 120%). This fractional perspective of percentages (Parker & Leinhardt, 1995) meant that students felt justified in simply finding one-fifth or 20% of the bar to ascertain the difference between that and the original bar. Students needed to appreciate here that the given bar actually represented 120% and that there are in fact six lots of 20% in the bar, not five. As a consequence, the bar needed to be divided by six to determine the weight of the extra 20%. This process took a considerable amount of time, effort and discussion for students to reach such conceptual understanding. Their initial ideas did not include making sense of a situation where thinking beyond 100 per cent was required.

Questions to support mathematical thinking

Current pedagogy suggests that asking questions to promote students' mathematics thinking is preferable to teachers 'explaining' a particular solution (Reinhart, 2000; McCosker & Diezman, 2009). The following questions were some that proved helpful to ask various groups during their exploration. They were designed to encourage students to reflect on their thinking processes and act as prompts for further probing of the mathematics ideas inherent in this task:

- When we look at this bar, what do we know about it?
- What mathematical information is it telling us?
- How can we draw a diagram of the bar to show what percentage we actually have here?

- How can we show the original chocolate bar in your drawing? What percentage is that representing?
- What does “20% extra” mean?

The last two questions appeared to be a significant challenge for many students because they were convinced that whatever whole (in this case, the bar with the 20% extra) was being dealt with, was equal to 100%. These students were operating on a fraction model that they had learned earlier where a percent could be used to describe part of a whole item. Percents greater than 100% did not occur within that paradigm and therefore made it difficult for them to appreciate the possibility of a ratio change.

When the students realised that the given bar actually represented 120%, they readily understood the logic of dividing 300 grams by 6 (which equalled 50 grams). This amount was deducted from 300 grams to find that the original bar weighed 250 grams. Students then checked their thinking by finding 20% (or one fifth) of 250 grams. They added that amount (50 grams) back onto 250 grams to get 300 grams. This represented the given bar of 120%.

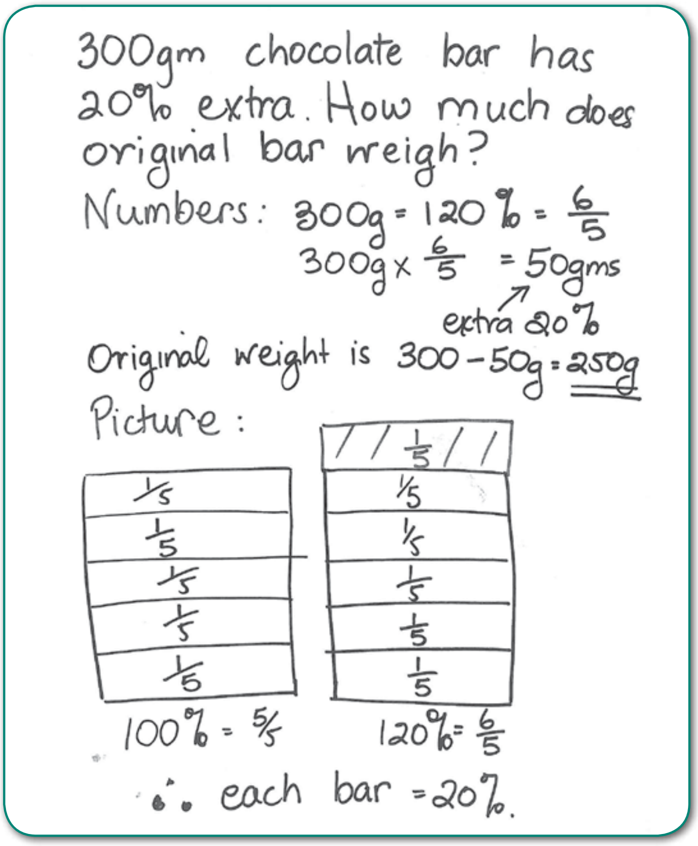

Figure 3

Figure 3 shows the drawing constructed by one group of students illustrating their revised thinking evoked by the posed questions. The use of a second drawing was a helpful strategy for reconceptualising the mathematics embedded in the problem. It appeared to support students to identify and consider the essential relationships between the chocolate bar, the number of grams and the percentage represented.

Observations

Using chocolate as a context was a drawcard that encouraged the students to engage with the task. Working with an item deemed to include “20% extra” gave them an opportunity to use fractional number ideas they had already learned. This included making the link between one-fifth and 20%. Students had some understanding of percentages, for example finding 20% or 50% of a given amount and were aware of numerous occasions when they had been exposed to percentages as consumers. This task was an opportunity for students to use reason and logic to consider and understand a particular use of percentages that is frequently used in their community. It showed that while learners can have sufficient computational skills to use fractional or percentage knowledge within contexts of 100%, they were not able to adjust their thinking to a situation outside that paradigm. However, the ability to question and prove the validity of such commonly displayed mathematical statements is fundamental knowledge and a right of any citizen in our society.

Working with ‘clumps” of percentages such as 20% or 50% “extra” initially, was a bridge to engage with a variety of percentage problems such as that illustrated here. To continue working with percentages students need to develop an understanding that an item may represent more than 100%.

A rich mathematical task

On reflection, this activity seemed to be a rich mathematical task (Ahmed,1987) for a number of reasons. It:

- was accessible in that the chocolate bar was a familiar context and learners had already had some experiences working with percentages;
- portrayed the use of mathematics in the community;
- provided an opportunity for learners to work together and discuss mathematics to develop their conceptual understanding;
- encouraged learners to use reason and logic to explain the mathematics involved;
- included an element of surprise when learners persevered with the mathematics to clarify their mathematical thinking.

Conclusion

Percentages are commonplace in our society. A greater understanding of how percentages work in situations requiring a ratio change would mean that people could make more informed decisions about the choices that are offered to them in everyday life. This study indicates that a rich task can expose misconceptions and provide opportunities for deeper learning. A chocolate bar may seem a somewhat frivolous context, but it proved to be a worthwhile opportunity for supporting students to develop their thinking about percentages greater than 100 percent.

Recommendations

- Provide a variety of situations requiring alternative ways of thinking to help students develop their knowledge and strategies regarding percentages.
- Present students with problems in contexts that are instantly recognisable.
- Introduce problems involving contexts of more than 100%.
- Provide students with opportunities to draw diagrams as a mechanism for exposing thinking and clarifying the mathematics.
- Ask key questions (avoiding direct explanations) about the mathematics embedded in the task to scaffold greater cognitive engagement.

References

Ahmed, A. (1987). *Better mathematics: A curriculum development study*. London: HMSO.

McCosker, N. & Diezman, C. (2009). Scaffolding students’ thinking in mathematical investigations. *Australian Primary Mathematics Classroom, 14*(3), 27–32.

Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research, 65*(4), 421–481.

Reinhart, S. C. (2000). Never say anything a kid can say. *Mathematics Teaching in the Middle School, 5*(8), 478–483.

Reys, R., Lindquist, M. M., Lambdin, D.V. & Smith, N. L. (2007). *Helping children learn mathematics*. John Wiley & Sons: New Jersey.

Stoessiger, R. (2002). An introduction to critical numeracy. *The Australian Mathematics Teacher, 58*(4), 17–20.

White, P., & Mitchelmore, M. (2005). Teaching percentage as a multiplicative relationship. In P. Clarkson, A Downton, D. Gron, M. Horne, A. McDonough, R. Pierce, A. Roche (Eds), *Building connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, pp. 783–790). Melbourne: MERGA.

Ngārewa Hāwera
University of Waikato
<ngarewa@waikato.ac.nz>
Merilyn Taylor
University of Waikato
<meta@waikato.ac.nz>

APMC